Ranking dynamics and volatility

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Joint work with

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Football competitions

- Recently Criado, Garcia, Pedroche and Romance (2013) studied rankings in European football competitions, trying to answer the question "Which competition is the most exciting", in the sense that there are many position switches in the rankings.
- They answered this question using competitivity graphs and derived measures of competitiveness.
- As we will apply their idea to any ranking, not just football competitions, and as the term competitiveness has a specific meaning in economics we will not use their term competitiveness but replace it by ranking dynamics, referring to the phenomenon of changes in rankings, mainly over time.

Our work

- In this contribution we will discuss the notion of ranking dynamics, consider how to measure it, look in more detail to the approach proposed by Criado et al. (2013), introduce a generalization and suggest applications.

The Criado et al. (2013) framework

- Consider a set E of n elements or nodes (when described in a network context), denoted as $\{e_1, e_2, \ldots, e_n\}$. Next we consider an ordered set **R** of rankings of these n elements. Rankings denoted as $c_1, \ldots, c_r$ are ordered (usually in time, referred to as instances), where each c_j is a complete ranking (no ties) of the n elements at instance j.

- We say that element e_i changes position with element e_j if they exchange their relative positions between two consecutive rankings. Roughly speaking the more position shifts the more dynamic a ranking system, e.g., a football competition.

An example

- We present a simple example: let n = 6 and let E = {e_1, e_2, …, e_6 }.

$$c_1 = (e_1, e_2, e_3, e_4, e_5, e_6)$$

$$c_2 = (e_1, e_4, e_6, e_5, e_2, e_3)$$

$$c_3 = (e_1, e_2, e_5, e_3, e_4, e_6)$$

$$c_4 = (e_4, e_2, e_3, e_1, e_5, e_6)$$

Different aspects when studying the dynamics of rankings

- 1) The underlying scoring method leading to a ranking.
- 2) Whether the ranking is complete or if ties are allowed.
- 3) The ‘timing’ of the r rankings.
- 4) One may study rankings of different entities as by Criado et al. who studied four football competitions; or rankings based on different criteria for the same entity (journals ranked by IF, immediacy index, total number of received citations).
- 5) Dynamics, see next slide.

Dynamics

- a) In the football case changes between consecutive rankings (weeks) are small as the maximum change in the underlying score is 3, but if one considers the final ranking at the end of the season then anything is (theoretically) possible.
- b) Another aspect is whether numbers on which rankings are based are completely independent between events (as for yearly final rankings in a national football competition) or are cumulative (as in the football competition data based on weekly or h-indices for researchers).
- c) Finally, another dynamic aspect is the fact that one must take into account that some teams/journals enter or leave the rankings.

Representing competitiveness in the sense of Criado et al.

- Criado et al. (2013) only studied the case of a fixed number of elements without ties in the ranking. We first describe their framework. They represent a set of rankings **R,** as a weighted network, with nodes $(e_j)_{j=1, \ldots, n}$. Two nodes are linked with weight k if these nodes perform k position shifts. If there are r instances, then there are at most r-1 position shifts (recall that position shifts are always considered between consecutive rankings or instances). We present a simple example: let n = 6 and let E = $\{e_1, e_2, \ldots, e_6\}$.

$$c_1 = (e_1, e_2, e_3, e_{4,} e_5, e_6)$$

$$c_2 = (e_1, e_4, e_6, e_{5,} e_2, e_3)$$

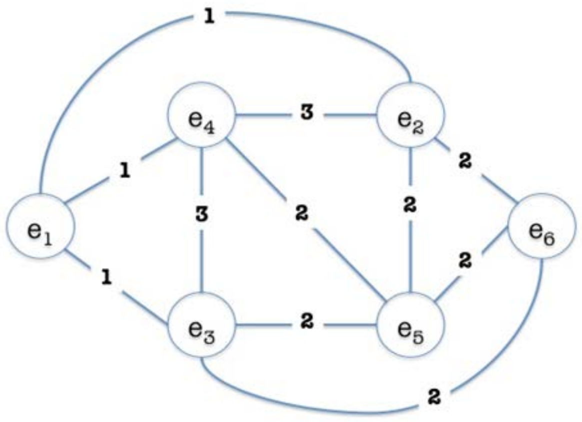
$$c_3 = (e_1, e_2, e_5, e_{3,} e_4, e_6)$$

$$c_4 = (e_4, e_2, e_3, e_{1,} e_5, e_6)$$

Matrix and network representations

$$c_1 = (e_1, e_2, e_3, e_4, e_5, e_6)$$
$$c_2 = (e_1, e_4, e_6, e_5, e_2, e_3)$$
$$c_3 = (e_1, e_2, e_5, e_3, e_4, e_6)$$
$$c_4 = (e_4, e_2, e_3, e_1, e_5, e_6)$$

$$\begin{array}{c} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{array} \begin{pmatrix} - & 1 & 1 & 1 & 0 & 0 \\ & - & 0 & 3 & 2 & 2 \\ & & - & 3 & 2 & 2 \\ & & & - & 2 & 0 \\ & & & & - & 2 \\ & & & & & - \end{pmatrix}$$



- The number of position shifts between two elements can be expressed in a (symmetric) matrix M or, equivalently, in a network.
- We see that the four possible cases: no shifts, 1, 2 and 3 shifts occur in this example. As a weighted network, called the ranking dynamics graph this leads to the network shown on the left.

Dynamics and volatility (Criado)

- Ranking dynamics (competitiveness in the case of (Criado et al., 2013)) is a property of an ordered set of rankings. The sum of all position shifts of an element is called its volatility. Using the matrix representation $M = (m_{ij})_{ij}$, the volatility of element e_j is defined as: $vol(e_j) = \sum_{i=1}^{n} m_{ij}$.

Measuring ranking dynamics (Criado)

- Absolute measure
- This measure is defined as the sum of the node strengths of all nodes in the ranking dynamics graph. This is twice the sum of all weights, or the sum of all elements in the matrix M.
- *Second measure: normalized form*
- The normalized mean strength, denoted as NS is the normalized sum of all node strengths in the ranking dynamics graph: NS(**R**) =

$$= \frac{\sum_{j=1}^{n} d_S(e_j)}{n(n-1)(r-1)}$$

Properties of measures of ranking dynamics (a)

- Axiom 1: Relabelling. Relabelling the elements (technicality: applying a permutation) must not change the resulting measure for dynamics. The dynamics is only determined by a change in overall configuration. This anonymity axiom must always be satisfied by a bona fide measure of ranking dynamics.

- Axiom 2: Replication. If the ordered set of r rankings, **R** = $(c_1, c_2, \ldots, c_r)$ is replaced by the ordered set of r+1 rankings **R'**= $(c_1, c_1, c_2, \ldots, c_r)$, where one of the c_j's (not necessarily the first) is duplicated, then this has no influence on absolute ranking dynamics. Of course, if more than one duplication occurs this has no influence either. A replication operation has no influence on the network and its weights. Note that duplication must occur between consecutive rankings. Replication must decrease relative ranking dynamics.

Properties of measures of ranking dynamics (b)

- Axiom 3: No change. If R = (c,c,c,c, …c) then there are no changes in rankings for consecutive instances, hence there is no dynamics. We require that a measure of ranking dynamics must take the value zero for this case. The “no-change” operation happens if one always applies alphabetical order.
- Axiom 4: Adding an element which is always ranked first or last. When this operation is applied there must be no influence on absolute measures but a decrease on relative measures of ranking dynamics.
- Axiom 5. Instance reversion. If the order of the instances is reversed then this has no influence on ranking dynamics

New entrants, leavers and ties

- In real-world rankings it often happens that new entrants join the set of elements or that some leave.
- Moreover, it may easily happen that ties occur. Hence, we will adapt the previous framework so that one can take new entrants, leavers and ties into account.

Notation (a)

- As in the Criado framework we consider a strictly ordered row of r instances. Each instance is a ranked set of elements, where ties are allowed. By definition, tied elements have the same rank. The symbol S denotes the set of all (different) elements occurring in the r instances; let #S = n. We **add** to each instance those elements in S which are absent in this particular instance, ranking them as ties on the last position. The original elements in a given instance are called the active elements, the added ones are called the inactive elements. Being active or inactive is referred to as the state of an element in a given instance. Active and inactive elements are separated by the symbol ; .

Notation (b)

- Denoting a tie between elements e and f as $\ldots,\underbrace{e,f},\ldots.$ and assuming that elements x, y and z are missing in instance c, this means that
- c = $(s_1, s_2, \ldots, s_{n-3})$ is rewritten as
- c = $\left(s_1, s_2, \ldots, s_{n-3}; \underbrace{x, y, z}\right)$
- The rank of an element s in instance c is denoted as $r_c(s)$.

Principles to calculate a dynamics indicator

- We will calculate the value of the ranking dynamics indicator NS.
- In its absolute form this indicator is the sum of the volatility scores of all elements in S. Besides position shifts, also leaving or entering, i.e. becoming active or becoming inactive are signs of dynamics and are taken into account.

Scoring (a)

- We obtain a volatility score for each element, e, by comparing with each other element.
- This score is obtained by a comparison of the positions and states of e and the other element in consecutive instances, leading to partial volatility score. The sum of all these volatility scores is the (global) volatility score of element e.
- Partial volatility scores are symmetric: the partial volatility score between elements e and f is the same as the one between f and e.
- In each comparison between instances the score either stays unchanged or increases by one. The initial score is zero.

Scoring (b)

- When the state of (at least) one of the two elements changes from active (A) to inactive (I) or vice versa then the score increases;
- if both elements do not change state, then we compare the relative position of e and the other element between two consecutive instances. If < changes to > or vice versa, then the volatility score increases; if there is no change in the relative ranking of e and the other element the score stays the same.
- If the two elements are active and become tied then the score does not change, but the previous relative position is kept in memory. If the two elements were tied and are still tied, then nothing changes; if they were tied and are not tied anymore then the last time they were not tied determines if there is a position change and hence if the volatility score increases or not.

An illustration

$$c_1 = (s,t,u,v)$$

$$c_2 = (s,u,w,x)$$

$$c_3 = (\underbrace{s,u},w,y)$$

$$c_4 = (z,y,u,s)$$

The set S is here {s,t,u,v,w,x,y,z}, with #S=8. Consequently the four instances are rewritten as:

$$c_1 = (s,t,u,v;\underbrace{w,x,y,z})$$

$$c_2 = (s,u,w,x;\underbrace{t,v,y,z})$$

$$c_3 = (\underbrace{s,u},w,y;\underbrace{t,v,x,z})$$

$$c_4 = (z,y,u,s;\underbrace{t,v,w,x})$$

Scoring: first steps

$$c_1 = \left(s,t,u,v;\underbrace{w,x,y,z} \right)$$

$$c_2 = \left(s,u,w,x;\underbrace{t,v,y,z} \right)$$

$$c_3 = (\underbrace{s,u},w,y;\underbrace{t,v,x,z})$$

$$c_4 = (z,y,u,s;\underbrace{t,v,w,x})$$

Now we illustrate, step by step, how we count the partial volatility score for {t,x}. We start with showing the initial position. Here, this initial position is: (t,A,x,I,c1,<,0,*). A first comparison yields:

(t,I,x,A,c2,>,1,*): the score becomes 1 because t becomes inactive (and moreover x becomes active, but this has no influence on the score anymore). Next we have:

(t,I,x,I,c3,=,2,>): x becomes inactive (leading to an increase in the score); we note that moreover t and x are tied and the last time they were not tied x was ranked before t (t>x)

(t,I,x,I,c4,=,2,>): this is the end result: the partial volatility score for the pair {t,x} is 2.

Complete results

elements	c_1-c_2	c_2-c_3	c_3-c_4	total		elements	c_1-c_2	c_2-c_3	c_3-c_4	total
s-t	0	1	0	1		u-w	1	0	1	2
s-u	0	0	1	1		u-x	1	1	0	2
s-v	1	0	0	1		u-y	0	1	1	2
s-w	1	0	1	2		u-z	0	0	1	1
s-x	1	1	0	2		v-w	1	0	1	2
s-y	0	1	1	2		v-x	1	1	0	2
s-z	0	0	1	1		v-y	1	1	0	2
t-u	1	0	0	1		v-z	1	0	1	2
t-v	1	0	0	1		w-x	1	1	1	3
t-w	1	0	1	2		w-y	1	1	1	3
t-x	1	1	0	2		w-z	1	0	1	2
t-y	1	1	0	2		x-y	1	1	0	2
t-z	1	0	1	2		x-z	1	1	1	3
u-v	1	0	0	1		y-z	0	1	1	2

The final result

- The global volatility of the elements in S are shown below. Clearly leavers and entrants have the highest volatility and hence contribute most to the overall ranking dynamics (also because in this simple example there are relatively many of them!).
- Finally, the total strength is 102 and NS, the normalized mean strength, is 102/(8x7x3) = 0.607.

Elements	s	t	u	v	w	x	y	z
Volatility	10	11	10	11	16	16	15	13

Stability

- In many application not the dynamics but the stability of rankings is of importance. Clearly, 1 – NS yields a relative measure of stability. In the previous example the corresponding stability measure, defined as 1 – NS, is 0.393.

An example:
JIF rankings (1997-2015)

JCR category	# Journals	r	NS(R)	
Economics	392	19	0.127	
Information Science & Library Science (LIS)	113	19	0.152	
Biology	138	19	0.130	

Thank you for your attention